

Inequalities

1 Axioms

For all real numbers a, b, c :

- Exactly one of $a < b$ or $b < a$ or $a = b$ is true
- $a < b$ and $b < c$ implies $a < c$
- $a < b$ implies $a + c < b + c$
- $0 < a$ and $0 < b$ implies $0 < ab$

Note, we define $a > b$ to be $b < a$.

2 Propositions

For all real numbers a, b, c, d :

- $a < b$ and $c < d$ implies $a + c < b + d$
- $a > 0$ iff $-a < 0$
- $a < b$ and $0 < c$ implies $ac < bc$
- $0 < a < b$ and $0 < c < d$ implies $ac < bd$
- $a < b$ and $c < 0$ implies $bc < ac$

The first three propositions were proved in class.

3 Integers

For integers, we add one more axiom:

- the smallest positive integer is 1