

Homework 15

Use the results of the preceding exercises to compute the sizes of the following sets of matrices.

1. $|GL_2(\mathbf{Z}_{26})|$
2. $|GL_3(\mathbf{Z}_{26})|$
3. $|GL_2(\mathbf{Z}_{95})|$
4. $|GL_2(\mathbf{Z}_{16})|$
5. $|GL_2(\mathbf{Z}_{17})|$
6. $|GL_2(\mathbf{Z}_{18})|$

Answers start on the next page. Don't peek until you have solved the problems.

Homework 15 - Answers

1. $|GL_2(\mathbf{Z}_2)| \cdot |GL_2(\mathbf{Z}_{13})| = (2^2 - 1)(2^2 - 2)(13^2 - 1)(13^2 - 13) = 157,248$
2. $|GL_3(\mathbf{Z}_2)| \cdot |GL_3(\mathbf{Z}_{13})| = (2^3 - 1)(2^3 - 2)(2^3 - 2^2)(13^3 - 1)(13^3 - 13)(13^3 - 13^2) = 1,634,038,189,056$
3. $|GL_2(\mathbf{Z}_5)| \cdot |GL_2(\mathbf{Z}_{19})| = (5^2 - 1)(5^2 - 5)(19^2 - 1)(19^2 - 19) = 59,097,600$
4. $2^{4(4-1)}|GL_2(\mathbf{Z}_2)| = 2^{12}(2^2 - 1)(2^2 - 2) = 24,576$
5. $(17^2 - 1)(17^2 - 17) = 78,336$
6. $|GL_2(\mathbf{Z}_2)| \cdot |GL_2(\mathbf{Z}_9)| = (2^2 - 1)(2^2 - 2)3^{4(2-1)}(3^2 - 1)(3^2 - 3) = 23,328$