Cyclic Subgroups and Subgroup Lattices

When viewing a group table in the Group Table Calculator, to the left of the table is a column of the group elements. Clicking on these elements selects/deselects them.

- If you have one group element selected and then click on the *Generate Subgroup* button, then the elements of the cyclic subgroup generated by that element will become highlighted.
- At the same time, the elements in the group table will be reordered so that the subgroup elements come first. When generating subgroups, you should be able to see that the subgroup is in fact closed under the appropriate operations.
- To look at other subgroups, there is a *Clear Selection* button which makes all group elements be not selected. Then click on one element and generate its subgroup.

Exercises:

1. For the following groups, generate each cyclic subgroup and make a note of what elements it contains. Keep track of when different elements generated the same subgroup. For example, if doing this for $\mathbf{Z}_6$, the answer would be:

$$\begin{aligned}\langle 0\rangle &= \{0\}\\ \langle 1\rangle &= \langle 5\rangle = \{0,1,2,3,4,5\}\\ \langle 2\rangle &= \langle 4\rangle = \{0,2,4\}\\ \langle 3\rangle &= \{0,3\}\end{aligned}$$

   (a) $\mathbf{Z}_n$ for $n = 1, 2, 3, 4, 5, 7, 8, 9$

   (b) S_3 (found in groups of order 6).

   (c) D_4 and Q_8 (found in groups of order 8).

2. Make a conjecture about cyclic subgroups in the the groups $\mathbf{Z}_n$ (note, proofs will come later).

3. If you are displaying a group table, you can click on the *Subgroup Diagram* button (just above the group table selectors). On the left side of the screen near the bottom is a button labeled *Show Diagram.* This will display the full subgroup diagram for the group. Note, there are some features to these diagrams we will address later in the course.

   Look at the subgroup diagrams for each of the groups considered above in #1.

4. Make a conjecture about the diagrams for groups $\mathbf{Z}_n$ for various n? You may want to first consider special cases like when n is a prime, or when it is a prime power, or when it is the product of 2 primes (note, proofs will come later).