Subgroups

1. Recall that a group is cyclic iff the whole group equals the cyclic subgroup generated by some element. In the group table calculator, determine if the following groups are cyclic by looking for a generator:

   (a) Mystery group $(6, 1000)$ (max order of 6, number 1000)

   (b) Mystery group $(6, 1002)$

2. For many questions, the answer will be easy for cyclic groups, or for abelian groups. Knowing some small examples of other groups inside and out is then very useful. From what we know about groups of small order, and then exploring other small groups answer the following.

   (a) determine the smallest two groups which are not cyclic

   (b) determine the smallest three groups which are not abelian

   (c) explain why every proper subgroup of the smallest non-cyclic groups are cyclic

   (d) explain why every proper subgroup of the smallest non-abelian groups are abelian

3. For each group, find a minimal generating set (of subsets which generate the group, one of the smallest order).

   (a) S_3 (in groups of order 6)

   (b) the five groups of order 8

   (c) the two groups of order 10

   Note, you can quickly check your work in each case by going to the subgroup diagram and clicking on the diamond representing the whole group. At the left, the page will give an example of a minimal generating set.

4. When looking at subgroup diagrams in the group table calculator, the subgroups are labelled by their orders. Look at the subgroup diagrams for the groups of order 12. What would you conjecture in general about orders of subgroups of finite groups?