# Name That Group

Being able to quickly determine the isomorphism type of a group from a group table reinforces ideas which we have been studying as far as intrinsic properties of groups.

1. Check the groups of orders up through 15, check that they can almost always be distinguished by

   - the number of elements in the group
   - the number of elements $g$ such that $g^2 = e$
   - whether or not the group is abelian

   What is the only pair of groups in this range which are not distinguished by these criteria?

2. In fact, for groups of order $\leq 15$, you usually do not need to check to see if the group is abelian. What are the only cases where you need to check if the group is abelian?

3. Identify the following groups (in terms of the names given for standard groups when listed by order).

   (a) Mystery group $(15, 11)$ (max order of 15, number 11)
   (b) Mystery group $(15, 13)$
   (c) Mystery group $(15, 18)$
   (d) Mystery group $(15, 21)$
   (e) Mystery group $(15, 65)$
   (f) Mystery group $(15, 69)$
   (g) Mystery group $(15, 113)$
   (h) Mystery group $(15, 204)$

4. From its group table, what group is $\text{Aff}(\mathbf{Z}_4)$.

5. What group is $S_3 \times \mathbf{Z}_2$?

6. In preparation of the next question, recall that if $S$ is a subset of a group $G$, its centralizer in $G$ is

$$C_G(S) = \{g \in G \mid gs = sg \forall s \in S\}.$$

   (a) Prove that $C_G(S)$ is always a subgroup of $G$.
   (b) Prove $C_G(\emptyset) = G$.

   Note, the *center* of $G$, $Z(G)$, is just $C_G(G)$. So, to compute the center of a group in the group calculator, select the group (or clearing its elements if it is already selected), and then click on the centralizer button twice.

7. From looking at the table of cosets (table of colors), identify $G/Z(G)$ for the following groups.

   (a) $G = D_4$

   (b) $G = Q_8$

   (c) $G = D_6$

   (d) $G = \mathbf{Z}_3 : \mathbf{Z}_4$

   (e) $G = \mathbf{Z}_8 :_3 \mathbf{Z}_2$ (among groups of order 16)

   (f) $G = \mathbf{Z}_2.D_4$ (among groups of order 16)