

Groups of Order 4 Homework

Suppose G is a group of order 4. Then G has an identity element, so call it e . Also, from an exercise, there exists $x \in G$ such that $x * x = e$ but $x \neq e$. Name such an element a , and let b and c denote the other elements of G .

- From the information so far about a and e , and the “sudoku rules” for group tables, fill in as much as you can for the group table of G :

$*$	e	a	b	c
e				
a				
b				
c				

You should have all but 4 squares filled in.

- Identify a open square which has only 2 possibilities for being filled in, give the two possibilities for it, and explain why it cannot be anything else.

- For each possibility from the last part, fill in the whole the group table.

Option 1: ____

$*$	e	a	b	c
e				
a				
b				
c				

Option 2: ____

$*$	e	a	b	c
e				
a				
b				
c				

- We know \mathbb{Z}_4 and V_4 are two non-isomorphic groups of order 4. Identify which table goes with which group (i.e., which is option 1 and which is option 2). For \mathbb{Z}_4 , give an explicit isomorphism $f : \mathbb{Z}_4 \rightarrow G$ where G is the group given by the corresponding option:

$$f(0) = \text{---} \quad f(1) = \text{---} \quad f(2) = \text{---} \quad f(3) = \text{---}$$